A STOCHASTIC MODEL
FOR DAILY RAINFALL DATA SYNTHESIS

By

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SYNOPSIS

A first-order Marcov Chain Model was utilized to synthesize the daily rainfall values observed at a point. Comparison was made between the historical and the synthesized daily rainfall values and they fit fairly well. These synthesized daily rainfall values will be used as input to the watershed systems model to produce daily synthetic runoff.

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INTRODUCTION

A stochastic process is defined as a collection of random variables $X(t)=[X_t,\ t\varepsilon T]$ which is a function of time and whose variate X_t is running along in time t within a range T. The set T is called the index set of the process. The stochastic process can be regarded as a discrete or a continuous process, depending on T. If T takes only discrete values, $T=[0,1,2,...]$ the process is termed a discrete process. If T takes continuous values $T=[t: -\infty<t<+\infty]$ the process is termed a continuous process. A value which X(t) takes is called a state of the process. The set of values in which all the values of X(t) lie is called the state space. (3).

Daily rainfall observed at a point is a continuously recorded hydrologic process. Analysis is performed by transforming the continuous process into a discrete process with time interval Δt. A real valued function defined on a sample space is called a random variable. Description of daily rainfall values as a discrete random variable is satisfactory when considering the recorded daily values, but it only approximates the natural rainfall process (8).

Let X_0, X_1, X_2 ... be the successive observations of daily rainfall values at times t = 0, 1, 2,...T. The possible values of X_t are 0.00, 0.01, 0.02, ... The collection of X_1, X_2, ... is referred to as rainfall process. Rainfall amounts observed during different short time intervals (hours, days) are not independent events. According to Grace and Eagleson (6) "There is sufficient information available in the literature to indicate that there is negligible dependence or serial correlation in series of annual rainfall depths. However, when the series of rainfall depths over shorter time intervals are analyzed, it is normally found that there is definite dependence inherent in the series."

Pattison (8) analyzed the successive hours of rainfall data observed at Boulder Creek, California, and showed the dependency of one hours rainfall to successive hours by estimating the conditional probabilities. Given, zero rainfall during hour, t

$$pr[X_{t+1} = 0.00/X_t = 0.00] = 0.962$$

$$pr[X_{t+1} = 0.01/X_t = 0.00] = 0.017$$

When the rainfall during hour t is 0.01 the conditional probabilities are

$$pr[X_{t+1} = 0.00/X_t = 0.01] = 0.397$$

and $pr[X_{t+1} = 0.01/X_t = 0.01] = 0.261$

He further states that the dependence is not confined only to consecutive hours of rainfall observations.

Gabrial and Newman (7) showed this dependence by estimating the conditional probabilities from successive days rainfall observation for Tel-Aviv, Israel, for the month of January.

pr[Wet day/previous day wet] = 0.674

pr[Wet day/previous day dry] = 0.293

pr[Dry day/previous day wet] = 0.326

and pr[Dry day/previous day dry] = 0.707

Wiser (10) has also shown daily dependencies for North Carolina stations. He states that dependency is found to be a quite general phenomenon. The degree of dependence is less for months than for days, is less for wet periods than for dry periods, and at some locations tends to a condition in which information about the previous day only is significant.

In order to check the dependence of one day rainfall to successive days rainfall, conditional probabilities were estimated from daily rainfall records for the month of July, observed at Bithlo, Florida. The estimated conditional probabilities are as follows:

pr[Wet day/previous day wet] = 0.517

pr[Wet day/previous day dry] = 0.327

pr[Dry day/previous day wet] = 0.483

and pr[Dry day/previous day dry] = 0.673

From the above calculated conditional probabilities estimates it can be concluded that the hourly and the daily rainfall process possesses the properties similar to that of the Marcov process. The Marcov process property states that the probability that a system will be in a given state at a given time, t, may be deduced from a knowledge of its state at any earlier time, t_0, and does not depend on the history of the system before t_0. A Marcov process with discrete parameter is called a Marcov chain.

A N^{th}-order Marcov chain model for a discrete stochastic process $[X_t,\ t = 0, 1, 2,\ldots]$ can be written mathematically as follows:

$$\begin{aligned} &\mathrm{pr}[X_t = x_t / X_{t-1} = x_{t-1}, \ldots, X_1 = x_1] \\ &= \mathrm{pr}[X_t = x_t / X_{t-1} = x_{t-1}, \ldots, X_{t-N} = x_{t-N}] \qquad (1) \end{aligned}$$

for all $X_1, X_2, \ldots, X_t$ and $t = N+1, N+2, \ldots$

A first-order Marcov chain model for (N=1) is written as:

$$pr[X_t = x_t/X_{t-1}, \ldots, X_1 = x_1]$$

$$= pr[X_t = x_t/X_{t-1} = x_{t-1}] \qquad (2)$$

If $X_{t-1} = i$ and $X_t = j$, then the system has made a transition from state i to state j at the t^{th} step. The probabilities of the various transitions that may occur is called the transitional probability and is written as:

$$p_{ij} = [X_t = x_t/X_{t-1} = x_{t-1}] \qquad (3)$$

The transition probabilities are estimated from the equivalent frequencies observed from the historic data. The frequency of occurence is obtained from the transition of processes from each of the states during time, t, to the same or other states in time, t+1. The frequencies can be arranged in the form of Table 1 in which f_{ij} represents the frequency of occurence of transitions between $(X = i)$ and $(X = j)$. The probability $\hat{p}_{ij}$ is estimated as:

$$\hat{p}_{ij} = f_{ij}/F_i \qquad (4)$$

$$F_i = \sum_{j=1}^{T} f_{ij} \qquad (5)$$

for $i = 1, 2, \ldots T$

and $j = 1, 2, \ldots T$

Table 1. Transition Frequencies

State I	State J 1	2	3	T	ΣF_i
1	f_{11}	f_{12}	f_{13}	f_{1T}	F_1
2	f_{21}	f_{22}	f_{23}	f_{2T}	F_2
.					.
.					.
.					.
T	f_{T1}	f_{T2}	f_{T3}	f_{TT}	F_T

Various researchers have applied the Marcov chain models for rainfall process analysis. Gabrial and Newman (7) were the first to apply the Marcov chain model to determine the occurence or non-occurence of rain on any day. They reported that the first-order Marcov chain model fitted well to frequency deduced from the daily rainfall observations. They were not concerned with synthesizing the rainfall depth values. The first reported research on synthesizing continuous sequences of hourly rainfall data was by Pattison (8). Pattison used the first and the 6th-order Marcov chain models. The first-order model was used for wet periods and the 6th-order model was used for dry periods. Pattison states that a first-order Marcov chain model fails to describe the transition between a sequence of wet hours and a sequence of dry hours because the occurence of an hour of zero rainfall at the end of wet hour sequence is considered by the process to be most likely the start of a sequence of dry hours, and in reality it is not so.

The problem which Pattison had with the in-between sequence while synthesizing the hourly rainfall values, does not arise with the daily rainfall values. Daily rainfall value is a point value and there is no in-between sequence. No reported research was available on synthesizing daily rainfall values. Many long records of daily rainfall values are available in comparison to hourly records, and these provide valuable information concerning the characteristics of the observation sites. The objective of this work is to set up a stochastic model for daily rainfall data synthesis. The synthesized daily rainfall data should duplicate the important statistical properties of the observed daily rainfall data. The synthesized daily rainfall values will be used as input to the watershed systems model being developed in-house. Daily rainfall data from Bithlo, Florida, was used to estimate the model parameters (transition probabilities).

Application of the Marcov Chain Model to Synthesize the Daily Rainfall Values for Bithlo.

Application of the first-order Marcov Chain Model (Equation 2) was made for the daily rainfall data synthesis for Bithlo. The synthesis procedure is exemplified by the detailed calculation of the model parameter estimation and the steps involved in synthesizing the daily rainfall depth values, for the month of July. Previously the conditional probability of the actual day being wet given the condition of the previous day (wet or dry) for the month of July was estimated. No rainfall depth assignments were made.

Ten years of daily rainfall values observed at Bithlo for the month of July are presented in Table 2. It can be seen from the table that the minimum observed daily rainfall value was 0.02 inch, and the maximum was 3.52 inches. If 3.52 inches is taken as the upper limit for the daily rainfall values, then $X(t)$, $t = 0.00, 0.01, 0.02, \ldots, 3.52$ can still take 352 different values. The probability of getting 3.52 inches of the daily rainfall value is very low. In order to reduce the daily rainfall process to take so many different values, daily rainfall values were grouped into 14 intervals as shown in Table 3. This will reduce the number of states which the model can take and at the same time eases the computational scheme. These 14 states now constitute the states of the first-order Marcov Chain Model, for the daily rainfall synthesis procedure for Bithlo. The daily rainfall process in terms of the Marcovian states are presented in Table 4. In terms of the first-order model, the process can pass from any of the 14 states from the previous day to any of the 14 states on the actual day. In other words, the size of the transitional probability matrix will be 14 x 14.

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Table 2. Daily Precipitation from Bithlo for the Month of July

DAYS	YEARS 1	2	3	4	5	6	7	8	9	10
1								.44		
2								2.45		.24
3	.64		.62		.06					
4	.22	1.30						.90	.65	
5		.91					.20			.57
6							.32		.70	.89
7		1.98		.30		1.44	.36		2.00	
8		3.40					.92			
9		1.82		.70	.52		3.58			.42
10	.37	.03	.58	.26			1.22			.28
11		1.00	.56				.13	.36		
12	.07	.30								
13	.08	.31		1.22			.65			
14				.05		.36	.20			
15	.33	.72		.47			.72		.15	
16	1.02	.20	.20			.22	.26	1.05		
17	.26				.37		.36		1.67	
18		.33	1.44	.49			.23	.85		.31
19	.05		.70	1.42			2.22			2.85
20	.98			.17						
21	.38				.72		.73			3.52
22	.56		1.00		.48	.40		2.82	1.22	.32
23		.60							.54	
24	.76	1.70				1.00		.40		
25		1.00				.60		1.05	1.48	
26	.02	1.50				1.90	.22	1.28		
27		.53		.49		.50				
28	.02	.48	.26	.25		.32			.08	.22
29		.31		.11		.35				
30	.10	.45		.42			.24	.07	.84	
31									.09	

TABLE 3 Interval Grouping of Daily Rainfall

Daily Rainfall State	Daily Rainfall Interval (Inches)
1	0.00
2	0.01-0.10
3	0.11-0.20
4	0.21-0.30
5	0.31-0.40
6	0.41-0.50
7	0.51-0.75
8	0.76-1.00
9	1.01-1.50
10	1.51-2.00
11	2.01-2.50
12	2.51-3.00
13	3.01-3.50
14	3.51-4.00

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Table 4. Daily Rainfall Conversion to Daily Rainfall State

DAYS	YEARS 1	2	3	4	5	6	7	8	9	10
								6		
1								11		4
2										
3	7		7		2			8		
4	4	9							7	
5		8					3			7
6							5		7	8
7		10		4		9	5		10	
8		13					8			
9		10		7	7		14			6
10	5	2	7	4			9			4
11		9	7				3	5		
12	2	4								
13	2	5		9			7			
14				2		5	3			
15	5	7		6			7		3	
16	9	3	3			4	4	9		
17	4				5		5		7	
18		5	9	6			4	8		5
19	2		7	9			11			12
20	8			3						
21	5				7		7			14
22	7		9		6	5		12	9	5
23		7							7	
24	8	10				9		5		
25		9				7		9	9	
26	2	9				10	4	9		
27		7		6		6				
28	2	6	4	4		5			2	4
29		5		3		5				
30	2	6		6			4	2	8	
31									2	

Table 5. Frequency of the Daily Rainfall Process For the Month of July at Bithlo

State During Day, T	State During Day, (T+1) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	ΣFi
1	120	9	3	8	8	3	14	4	9	1	0	1	0	1	181
2	7	1	0	0	0	1	0	1	1	0	0	0	0	0	11
3	3	0	0	0	2	1	1	0	0	0	0	0	0	0	7
4	11	0	1	0	2	0	0	0	0	0	1	0	0	0	15
5	8	0	0	1	2	1	1	1	1	0	0	1	0	0	16
6	4	0	0	2	3	0	0	0	1	0	0	0	0	0	10
7	6	0	1	2	0	1	1	1	0	3	0	0	0	0	15
8	5	1	0	0	1	0	0	0	0	0	0	0	0	1	8
9	4	1	2	2	0	0	4	1	2	0	0	0	0	0	16
10	1	1	0	0	0	1	0	0	1	0	0	0	1	0	5
11	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
13	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
14	0	0	0	0	1	0	0	0	0	1	0	0	0	0	2

TABLE 6. ESTIMATES OF CUMULATIVE TRANSITION PROBABILITY FOR THE DAILY RAINFALL PROCESS DURING JULY AT BITHLO

State during day, T

State during day, (T+1)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.680	.731	.743	.783	.829	.847	.927	.945	.985	.990	(.992)	.995	(.998)	1.000
2	.636	.727	(.748)	(.769)	(.790)	.818	(.862)	.909	1.000	1.000	1.000	1.000	1.000	1.000
3	.428	(.499)	(.570)	(.643)	.714	.857	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	.714	(.749)	.785	(.877)	.929	(.941)	(.953)	(.965)	(.977)	(.989)	1.000	1.000	1.000	1.000
5	.500	(.521)	(.541)	.562	.690	.752	.814	.876	.938	(.959)	(.979)	1.000	1.000	1.000
6	.400	(.465)	(.530)	.600	.900	(.925)	(.950)	(.975)	1.000	1.000	1.000	1.000	1.000	1.000
7	.400	(.433)	.467	.599	(.632)	.666	.733	.800	(.900)	1.000	1.000	1.000	1.000	1.000
8	.625	.750	(.791)	(.835)	.875	(.889)	(.903)	(.917)	(.931)	(.945)	(.959)	(.973)	(.987)	1.000
9	.235	.294	.411	.528	(.607)	(.686)	.766	.883	1.000	1.000	1.000	1.000	1.000	1.000
10	.200	.400	(.450)	(.500)	(.550)	(.600)	(.665)	(.730)	.800	(.850)	(.900)	(.950)	1.000	1.000
11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
13	(.100)	(.200)	(.300)	(.400)	(.500)	(.600)	(.700)	(.800)	(.900)	1.000	1.000	1.000	1.000	1.000
14	(.100)	(.200)	(.300)	(.400)	(.500)	(.600)	(.700)	(.800)	(.900)	1.000	1.000	1.000	1.000	1.000

Monte-Carlo simulation technique; i.e., random sampling, was used to generate the daily synthetic rainfall data. The procedure was programmed for IBM 1130. A flow chart is listed in the appendix.

1) For the synthesis of the rainfall process for day, t+1, determine the state of the previous day, t. It can be assumed that the state of the previous day, t, is dry.
2) State of day, t+1, is selected at random by using the estimated probabilities that determine the transitions from a dry state to either a dry or wet state in day, t+1.
3) If the state of day, t+1, is determined to be dry, the synthesis moves on to the next day.
4) If the state of day, t+1, is determined to be wet then the magnitude of X(t+1) is selected using the transition probability. Process is terminated for day, t+1.
5) If the state of day, t, is found to be wet [X(t)=2,3...14] the state of day, t+1, is selected at random using estimates of the probabilities. After the selection of X(t+1), the procedure moves on to the next day.
6) The state of the rainfall system for each day has to be transformed into a rainfall amount in inches. Mid-point values of the rainfall intervals listed in Table 3 were used as the rainfall amounts for each state.
7) Different probability estimates of daily rainfall were used for different months to take into account the seasonal variability.
8) Repeat steps 1 through 7 for as many years of synthesized daily rainfall values as desired.

RESULTS AND DISCUSSION

Twenty years of daily rainfall values were synthesized for Bithlo by use of the first-order Marcov chain model. Monthly historic values and the synthesized values are presented in Tables 7 and 8. In order to check the adequacy of the first-order Marcov chain model to represent the daily rainfall process for Bithlo, the Kolmogorov Smirnov two-sample test was used for the month of July. This test is used to test whether the two samples, i.e., the samples from the historic and synthesized data, have been drawn from the same population. If they are drawn from the same population, then their cumulative frequency distributions should show only random deviations from the distribution of the population.

To apply the test, cumulative frequencies were derived from the historic and the synthesized states. An α value of 0.01 level of significance was used for the test. The computed cumulative frequencies from the historic and synthesized states are presented in Table 9. They are also plotted in Figure 1. The largest absolute difference between the two distributions is the test statistic, D.

$$D = \mathrm{Max}/S_h(X) - S_s(X)/$$

where $S_h(X)$ and $S_s(X)$ are the cumulative frequency distributions for the historic states and the synthesized states.

Table 7. 20 years of synthesized daily rainfall values summed together for months.

Run NO.	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sept.	Oct	Nov	Dec
1	2.8	4.7	4.0	1.3	2.0	6.5	6.2	5.5	4.5	2.5	0.7	2.0
2	3.0	6.7	7.1	1.2	2.0	18.0	11.4	12.4	10.4	4.9	1.5	1.8
3	5.3	7.1	7.5	3.0	2.9	10.7	7.6	8.3	8.0	4.0	2.2	5.1
4	0.2	1.5	1.7	0.0	0.2	5.7	5.5	3.7	12.5	1.4	0.0	0.1
5	0.2	1.2	0.2	0.9	0.1	6.7	4.0	12.2	11.5	0.3	0.0	0.1
6	1.7	3.7	2.7	1.9	7.7	4.7	7.5	7.0	5.3	2.7	1.6	1.1
7	1.5	2.9	3.2	1.2	2.7	3.7	5.8	15.1	4.4	1.8	0.7	2.1
8	2.3	5.8	2.7	1.3	1.3	9.0	10.2	8.8	5.5	2.5	1.5	1.7
9	2.0	4.6	3.7	1.7	9.7	9.5	8.6	7.1	4.8	2.6	1.5	1.1
10	3.1	6.0	6.0	0.8	2.5	12.2	10.9	12.1	9.9	5.0	1.8	2.9
11	1.8	4.0	2.4	1.0	7.0	7.0	6.7	6.3	5.1	2.7	0.7	2.3
12	1.5	2.4	3.0	1.2	7.5	5.2	6.3	4.3	5.1	2.0	1.6	0.8
13	1.5	2.7	2.8	0.1	0.9	5.2	7.4	6.3	13.4	1.6	0.7	0.8
14	0.5	2.2	1.5	1.2	0.2	4.5	4.7	13.5	3.2	1.3	0.7	0.2
15	3.5	4.7	3.5	1.6	1.6	9.7	7.3	7.3	4.2	3.0	0.7	2.2
16	3.8	5.5	4.5	0.1	7.7	6.7	10.1	8.3	14.9	3.3	1.5	2.3
17	0.7	2.5	2.2	1.7	0.3	9.2	8.5	4.8	5.1	1.4	1.5	0.2
18	3.1	7.2	4.0	1.5	6.6	9.7	10.2	11.8	14.8	3.5	0.7	2.9
19	2.0	5.5	2.3	1.5	0.8	17.0	9.4	7.3	9.0	1.6	2.2	2.0
20	1.2	2.5	3.8	0.3	0.8	4.7	6.3	15.0	5.0	1.1	0.7	0.4

Table 8. 10 years of historic daily rainfall values summed together for months.

Year	Jan	Feb	Mar.	Apr.	May.	June	July	Aug.	Sept.	Oct	Nov	Dec.
1	3.65	4.18	8.00	4.07	3.03	9.60	5.92	5.47	8.50	5.52	1.20	2.60
2	1.08	4.22	13.23	1.19	1.55	11.34	19.87	5.68	12.10	2.85	.25	.68
3	2.02	2.25	1.73	.25	2.23	5.99	4.37	11.65	13.00	1.40	1.26	1.43
4	1.23	2.43	3.06	1.60	.36	3.40	6.51	12.01	8.28	1.51	2.42	1.38
5	2.22	3.61	4.42	.76	7.30	10.77	2.15	9.03	9.77	.24	9.06	2.44
6	5.16	3.12	2.60	2.80	3.77	8.92	7.09	14.00	7.02	2.33	.37	.98
7	1.68	4.34	3.62	.74	.09	4.05	12.56	10.61	4.63	4.10	.00	3.92
8	4.47	5.90	2.45	1.74	5.66	10.01	11.67	3.42	15.02	1.70	.44	.53
9	1.51	4.88	1.08	.00	.94	11.60	8.42	10.06	6.86	.00	.00	2.86
10	.70	7.77	2.29	1.50	6.07	16.98	9.62	8.91	2.80	9.25	2.56	.55

Table 9. Cumulative Frequency of the Historic and the Synthesized Daily Rainfall States for the Month of July, for Bithlo, Florida.

State	Hist. Cum. Freq.	Syn. Cum. Freq.
1	.611	.601
2	.649	.652
3	.675	.674
4	.724	.723
5	.783	.794
6	.814	.849
7	.882	.930
8	.908	.952
9	.963	.979
10	.979	.982
11	.985	.985
12	.991	.997
13	.994	1.000
14	1.000	1.000

$$D = /.882 - .930/ = .048$$

The tabulated value of D_{cr} at 0.01 level of significance (4) is .094. As the calculated D value is lower than the tabulated D_{cr} value, it can be said that the first-order Marcov model adequately represents the daily rainfall process for Bithlo.

The storm length from the synthesized and the historic daily rainfall values were also subjected to Kolmogorov-Smirnov two-sample test. The tabulated D_{cr} value at 0.01 level of significance is 0.163 (sample size 100) and the maximum absolute computed value is .120. It can be said that the storm lengths are also significant at 1% level.

X^2 test was used to test the frequencies derived for the number of wet days, from the historic and synthesized daily rainfall values. Calculated X^2 values together with the table value for 0.01 level of significance is presented in Table 8.

Table 10. X^2 Test Statistics for the Frequencies of the Number of Wet Days.

Month	X^2 Calc.	X^2 .99
January	.3020	6.63
February	.0080	
March	.0430	
April	.1560	
May	.3430	
June	.0250	
July	.0008	
August	.0770	
September	.0560	
October	.2430	
November	.1630	
December	.0140	

All of the above tests, (Kolgomolov-Smirnov and X^2), indicate that the first-order Marcov chain model is adequate for daily rainfall synthesis procedure. However, Franz (5), states that the application of statistical tests is hazardous because the assumption of random sampling is often violated. He states further that personal judgment based on experience and tempered by rough statistical calculations should be given more weight than the so-called "precise and powerful" normal theory tests.

Comparison of the monthly means, maximum and minimum values, and the average number of wet days, have also been made from the historic and synthetic data. These values are presented in Tables 11 and 12.

From Table 11 and Figure 2, it can be seen that the historic and the synthesized means match fairly well, except for the month of September. The difference between the two means for this month is more than 3.5 inches. In general the synthesized mean monthly values are lower than the historic means.

Table 11. - Statistical Properties Comparison Between Synthesized and Observed Rainfall

MEAN	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
HIST.	2.37	4.28	4.24	1.46	3.10	9.26	8.82	9.08	8.80	2.90	1.75	1.68
SYNT.	2.08	4.17	3.44	1.18	2.92	8.28	7.73	8.85	5.37	2.46	1.12	1.60
H. MAX. VALUE	5.16	5.90	13.23	4.07	7.30	16.98	19.87	14.00	15.02	9.25	9.06	3.92
S. MAX. VALUE	5.03	7.10	7.50	3.00	9.70	18.00	11.40	15.10	14.90	5.00	2.20	5.10
H. MIN.	0.70	2.25	1.08	0.25	0.09	3.40	2.15	3.42	2.80	0.24	0.25	0.53
SYNT.MIN.	0.20	1.20	0.02	0.10	0.10	4.50	4.00	2.20	1.50	0.30	0.00	0.10

Table 12. - Comparison of Average Number of Wet Days in a Month

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
HIST. WET	4.0	4.8	5.8	2.3	5.7	9.7	12.1	10.4	8.7	4.1	2.2	2.8
SYNT. DRY	2.9	4.6	5.3	1.7	4.3	9.2	12.2	9.5	8.0	5.1	1.6	2.6

Comparison of the historic and synthesized maximum and minimum values are also presented in Table 11. For the months of March, July, October and November, historical maximum values are higher than the synthesized maximum values. For the months of February, May, June and December, the synthesized maximum values are higher than the historic values. It could be due to the fact that the synthesized values are the mid-point values which are almost static. Maybe, rather than assigning the mid-point values as such, if a random component is added to it, then the assignment of rainfall depth values will be more flexible and the differences between the historic and the synthesized values will be minimized.

The historical and synthesized minimum values match fairly well except for the months of March, June, July, August and September.

Comparison of the average number of wet days in a month is presented in Table 12 and Figure 3. June, July, August and September are the rainy months in Florida and for these months the match between the historical and synthesized number of wet days matches fairly well. For the rest of the months there are some discrepancies.

Concluding Remarks

Several models are under study by the Central and Southern Florida Flood Control District in a continuing search for optimal management and effective control of water resource systems. These models comprise the a) Watershed systems model, b) Economic model, and c) Rainfall model (Figure 4).

The rainfall model discussed in this paper was developed with the intention of providing synthetic input data to the watershed systems model. So far, the actual application of this model has not been made.

Acknowledgement

The continuing support and interest of our Chief Engineer, Mr. W. V. Storch, is appreciated very much. Without his encouragement and guidance, this work would not have reached this point. This work is partly supported by the U. S. Water Resources Grant No. 101 of Public Law 88-379.

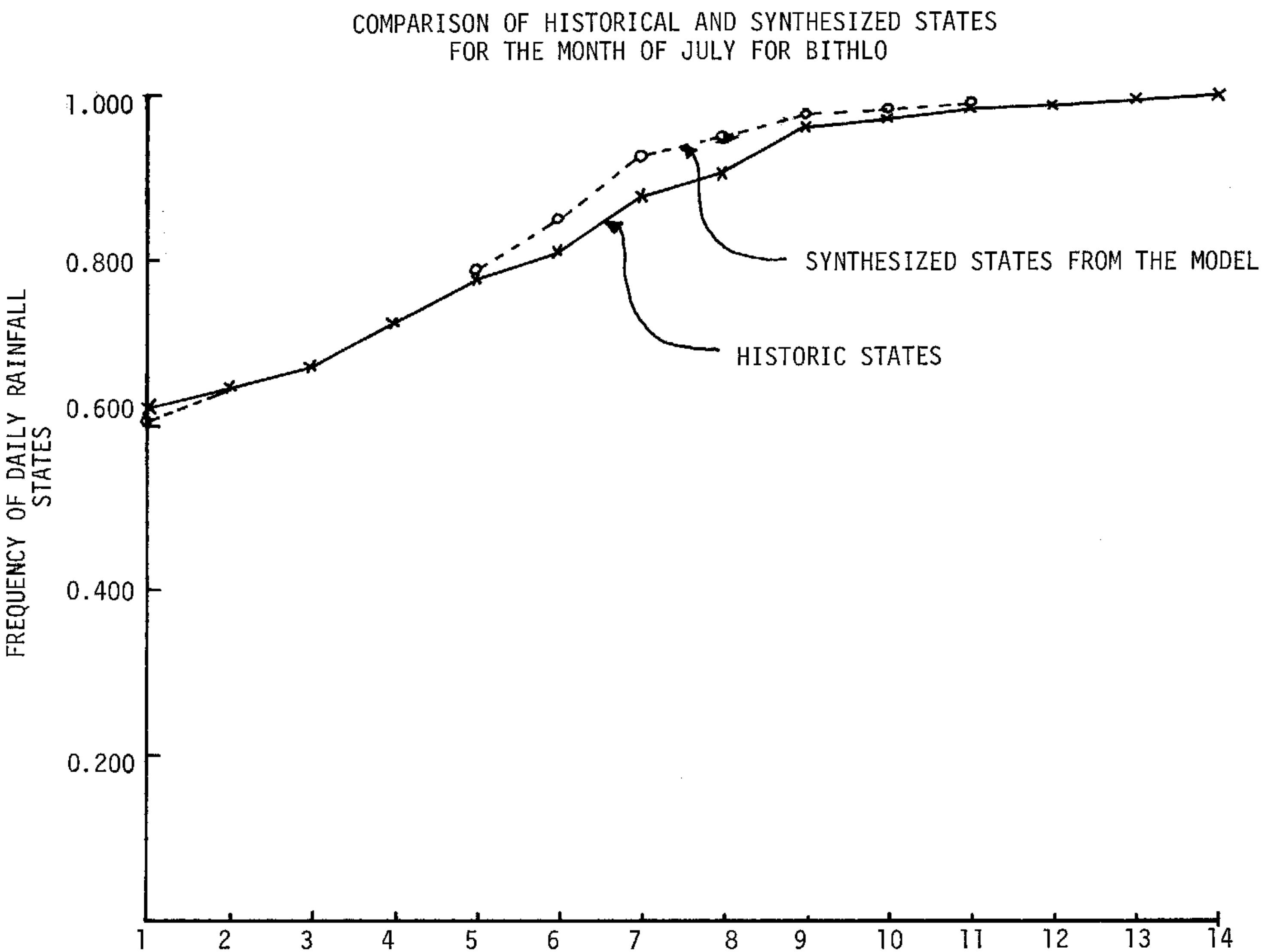


Figure 1 - STATES

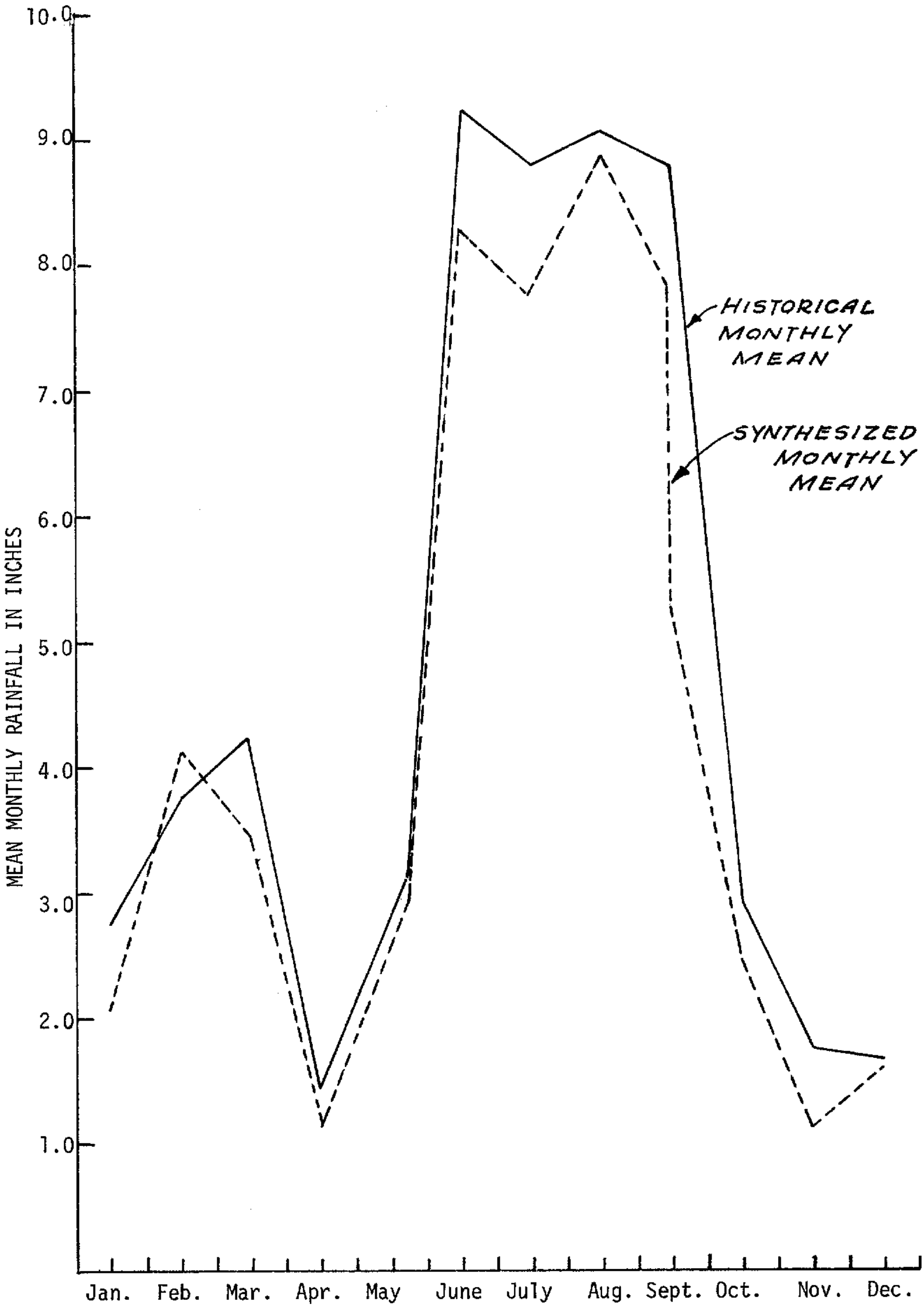


Figure 2. - MONTHS

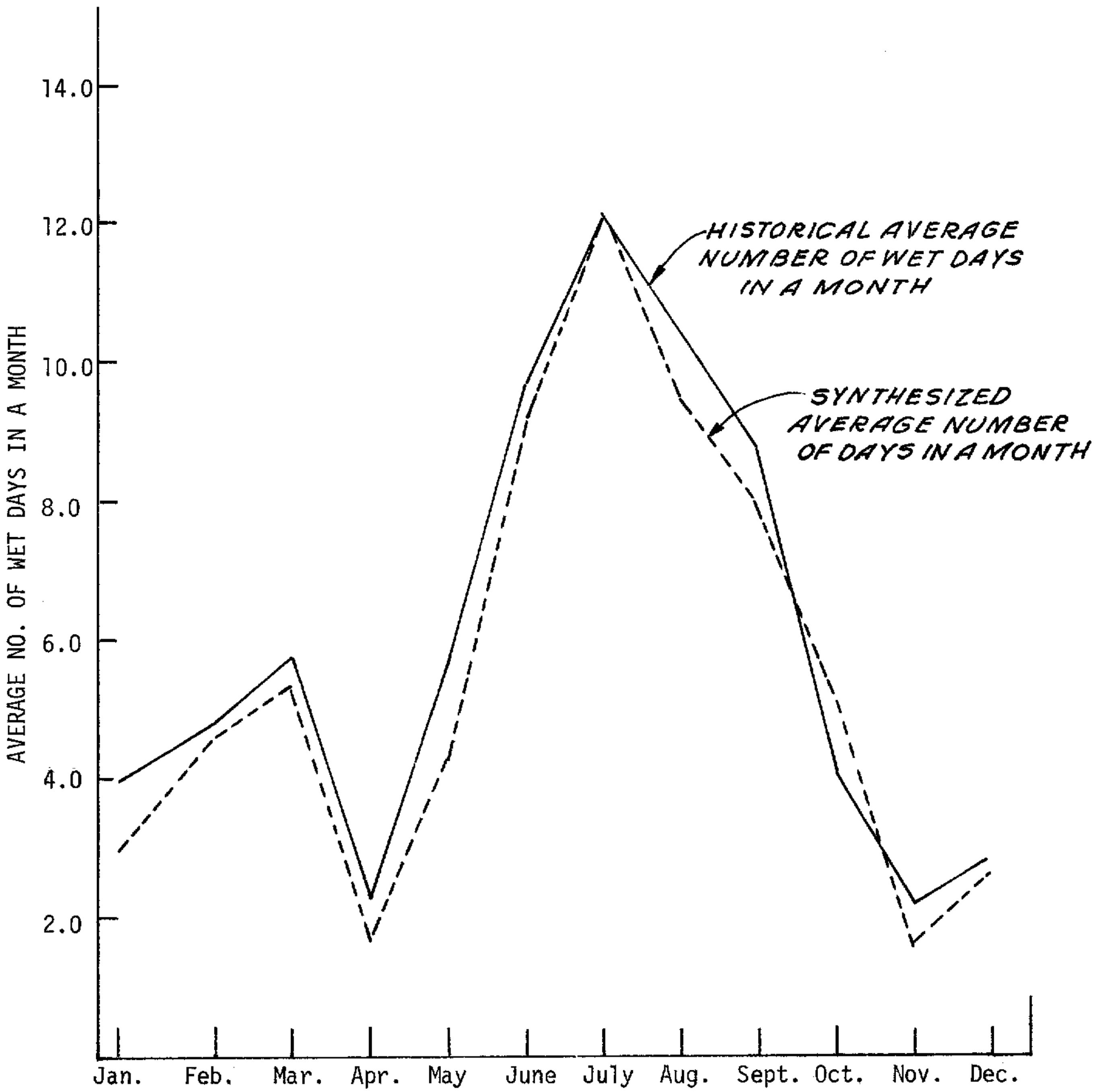


Figure 3 - MONTHS

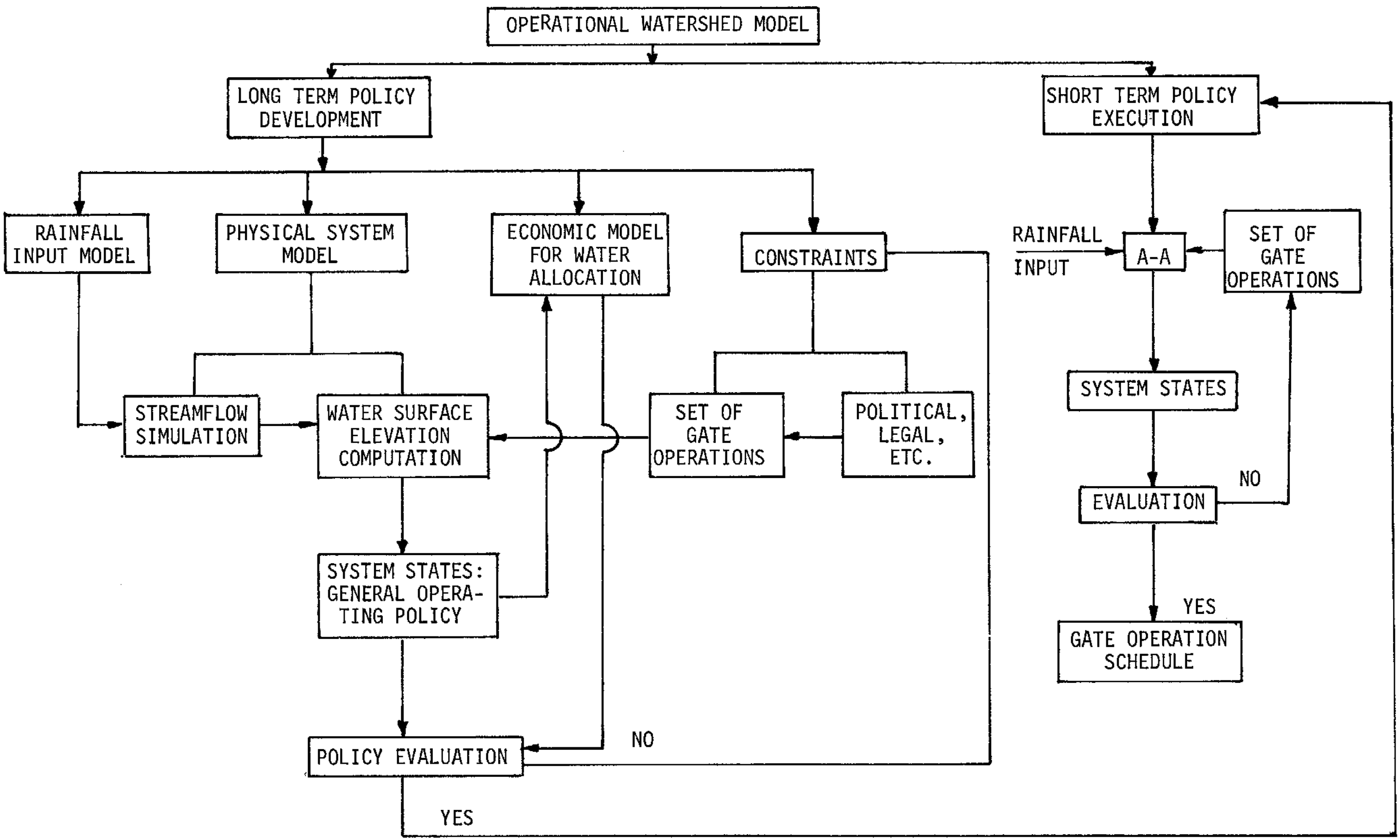
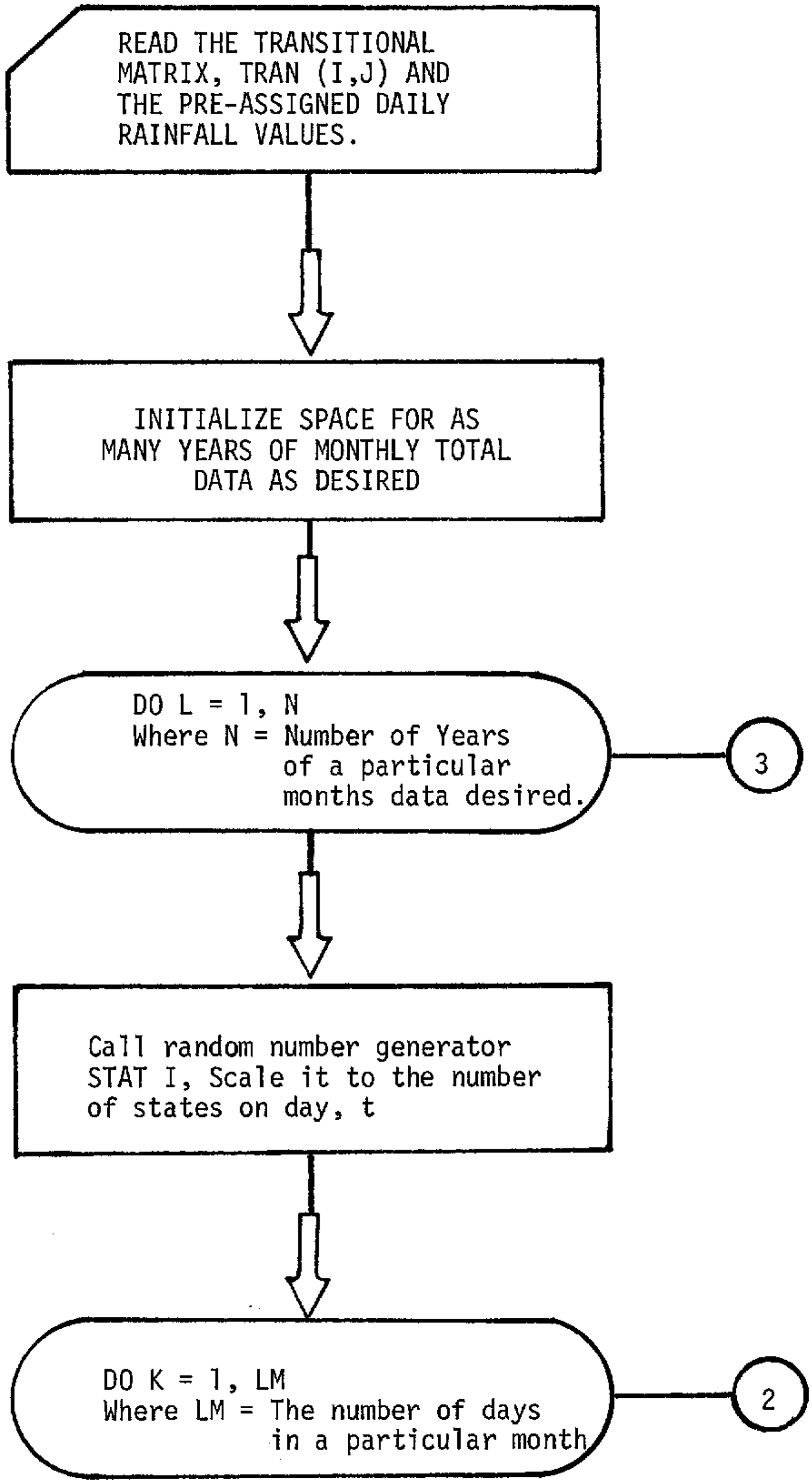
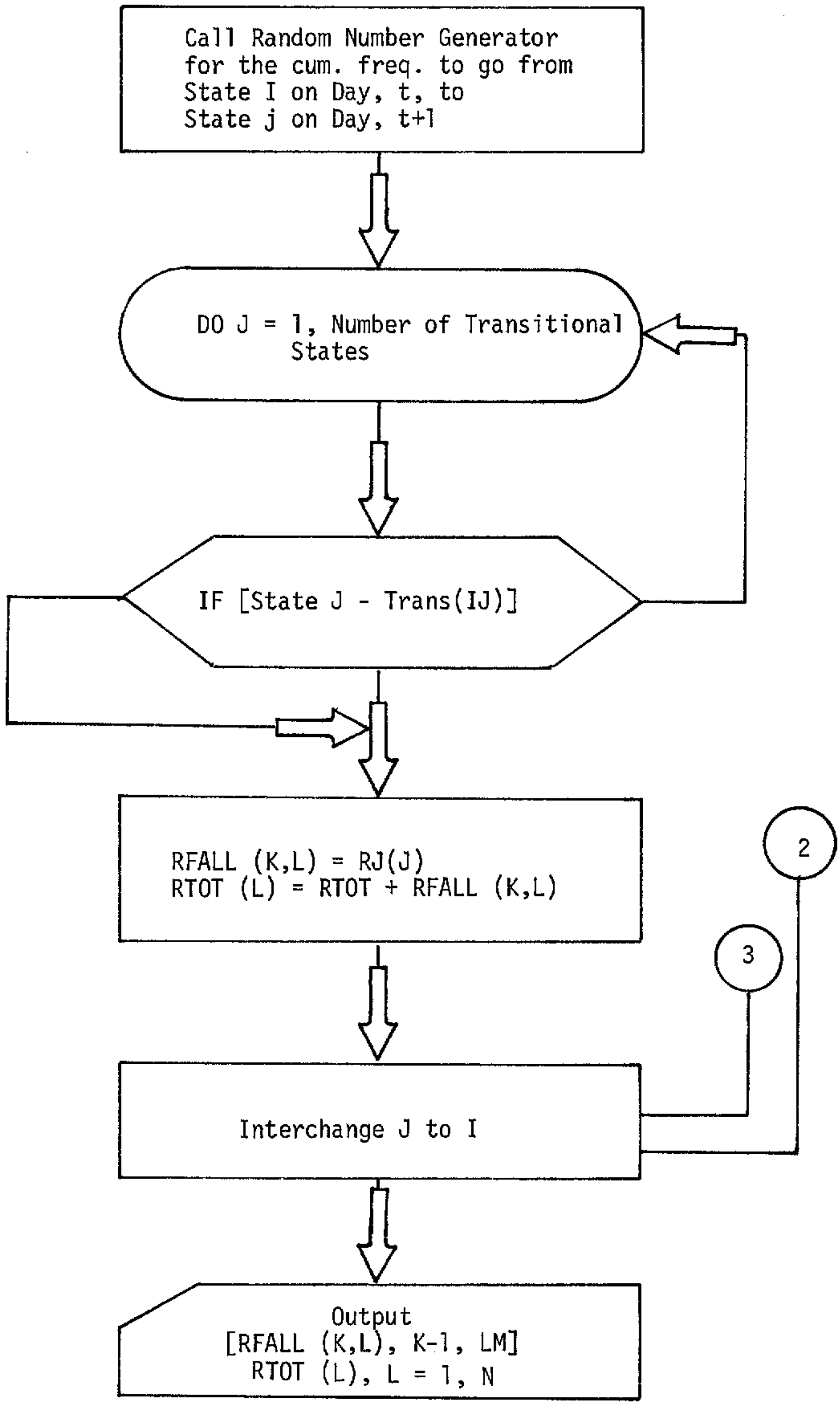


Figure 4 - SCHEMATIC OPERATIONAL WATERSHED MODEL

APPENDIX

FLOW CHART FOR SYNTHESIS PROCEDURE




Call Random Number Generator
for the cum. freq. to go from
State I on Day, t, to
State j on Day, t+1
DO J = 1, Number of Transitional States
IF [State J - Trans(IJ)]
RFALL (K,L) = RJ(J)
RTOT (L) = RTOT + RFALL (K,L)
2
3
Interchange J to I
Output
[RFALL (K,L), K-1, LM]
RTOT (L), L = 1, N

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